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Research Article

Theoretical and simulation of central elliptical hole with rectangular plate

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Abstract. A study on design engineering components with slot, notches is very important, because there is a stress increases/failure region area, where the force/stress is concentrating more and more. The elastic stress concentration mainly depends on the mode of loading, materials, and geometry of the design engineering components. The design engineers, academicians, and researchers concentrated and focused on fail-safe design and safe life design. A Plate is considered with different slots, such as circular and elliptical. The main objective of this study is to find out the stress concentration factor in plates with various cutout shapes. This concept is used in design components/structures, for finding the elastic stress concentration. The methods compared are tabulated with their findings. Singularities of the circular hole and elliptical hole in rectangular plates are considered in the present study. The finite Element Method (FEM) was used for fine mesh and ANSYS WORKBENCH software was used for extracting the results and results were validated by analytical or experimental methods.

Keywords: Metal cutting; Ansys workbench; Aspect Ratio; stress concentration; Von-Mises stress

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1. Introduction

In this work stress concentration depends on various parameters such as type of cutout shape, dimensions of plates, and loading condition (Kawadkar & Bhopale, 2012). Few authors studied stress concentration, focusing on the structural members that are mostly subject to (or weakened by) stress concentration (Goo et al.,2012). Among such studies, the work on stress concentration on cutouts includes analytical works for the plates with various cutout shape (Savin et al.,1961), optimum design of holes and different cutouts by considering fatigue life (Won et al.,2008). Stress concentration around irregular holes using complex variable methods is reported by (Simha et al., 1998). Conformal mapping method has been used for the evaluation of stresses. The method is an operation in complex mathematics that maps a set of points in one coordinate system to a corresponding set in another, keeping the angle of intersection between two curves constant, and is widely used in solving elasticity problems. Nine-hole shapes with the same area and different perimeters are studied. Irregular holes may change their shape if not their size by exchanging surface energy with strain energy. A linear elastic analysis followed in this paper may not support the physics of change in shapes but can be extended to a linear visco-elastic material. The analytical solution of an infinite elastic plate with a circular hole and an elliptic hole subjected to arbitrary biaxial loading is obtained by (Xin-Lin Gao et al.,1996). The complex potential method is used to formulate the boundary-value problem, and the elliptic-hyperbolic coordinate system is adopted to simplify the formulation. Two adjustable parameters-the biaxial loading factor λ and the orientation angle β-contained in the expressions for stress and displacement field are derived.

It should be pointed out that in his analysis the hole boundary is assumed to be traction-free. In the formulation given by (Savin et al.,1961) for stresses around holes in anisotropic plates under in plane loading a general solution is obtained to consider an arbitrary shape of the hole and arbitrarily oriented uniaxial, biaxial, and shear stresses at infinity as well as uniform tangential force, and uniform pressure around the hole. This is achieved by (Ukadgaonker and Rao,1993) introducing a general form of mapping function and an arbitrary biaxial loading condition into the boundary conditions. They extended the basic formulation for multilayered plates. They introduce the computer program, the constants of the mapping function, the arbitrary biaxial loading factor, the orientation angle, and the complex parameters for the laminate, the stresses around the hole can be easily (Zirka et al.,1993) have analyzed stress concentration around a circular hole in a rectangular plate for orthotropic and isotropic plates under dynamic and static loading. They have used the photoelastic method for analysis (Ukadgaonkar et al.,1994) have analyzed the stresses in an orthotropic plate with an irregularly shaped hole for different in-plane loading. They find the solutions for in-plane loading that are applicable for isotropic as well as for orthotropic cases. They found that angle ply laminates are inducing more

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stress concentration for shear loads and are not suitable. They produce many results with different shapes of holes for uni-axial, (loading in x-or y-y-direction), as well as for biaxial loading conditions, and also a pure shear loading case is considered. These results are supported by finite element solutions.

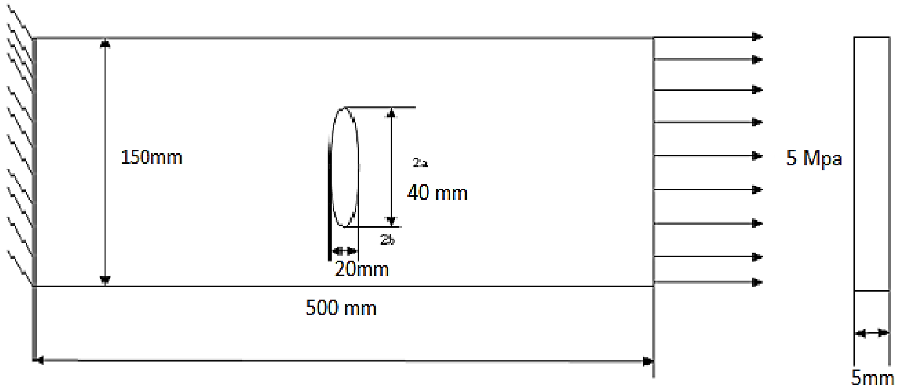


Fig. 1 Geometry of static structural plate with a central elliptical hole

Using (Muskhelishvili's et al.,1996) complex variable method—a very potent and elegant tool to solve two-dimensional problems of elasticity—Savin et al.,1961] and (Lekhnitski et al.,2008) present an extensive literature on exact solutions of stress field around holes of different shapes in infinite isotropic and anisotropic plate for uniaxial, biaxial, and shear edge loadings. (Savin et al.,1961) (Ukadgaonker et al.,2000), (Zirka et al.,1993), and (Ukadgaonker et al.,1994) employed the Schwarz alternating technique to get, by consecutive approximations, the stress distribution surrounding holes of different shapes (circular, elliptical, triangular, and rectangular) with rounded corners. Stress concentration around irregular holes was found by Simha and Mohapatra [Simha et al., 1998] using a complex variable technique. Sharma (Sharma et al.,], Chauhan describe the stress field surrounding polygonal holes in infinite and finite plates for various material qualities and loading circumstances. Patel and Sharma (Patel et al., 2017), and Chauhan (M. M. Chauhan et al.,2016). The literature (Shahzad et al.,**2019)**,(Shahzad et al.,2018), and also takes into consideration the analytical solutions for voids and inclusions embedded in planes exposed to out-of-plane loading. Within strain gradient and pair stress elasticity, problems with comparable settings are studied (Khakalo et al.,2016), (Lubarda et al.,**2016**). The advantage of progressive spatial variation in elastic properties is exploited by functionally graded materials. Numerous researchers have found the solutions of stress concentration around different shaped holes in functionally graded (FG) plates (Yang et al.,2010), (Mohammadi et al.,2010), (Mohammadi et al.,2011), (Kubair et al.,2007), and (Enab 2014).

There has been little work documented on the impact of loading that varies linearly. Most research focuses on the analysis of stress distribution around holes of different shapes in rectangular plates subjected to uniaxial, biaxial, and shear loadings. Few issues with plates buckling Loads subjected to linearly varying in-plane loads are available in the literature (Komur et al.,2008], (Kadkhodayan et al.,2014), (Aydogdu et al.,2018), (Kumar et al.,2015). Kawadkar (Kawadkar et al.,2012) presented an elastic solution for a large rectangular plate with a circular hole in the centre and subjected to linearly varying in-plane stresses at two opposing edges. he introduced the load factor Avoid overlays. By choosing the appropriate load factor Different types of loading conditions can be applied.

2. Materials and methods

In the present study, the effect of aspect ratio (b/a) of a central elliptical hole on the stress distribution and deflection in a rectangular 3D plate of dimensions 500mm x 150mm x 5mm under longitudinal static load of magnitude 5Mpa has been analyzed using finite element method as shown in Fig.1. Due to the presence of elliptical hole in the plate the maximum equivalent Von-Mises stress induced is expected at the corner of major axis of the hole. Stress concentration is defined as the localization of high stresses due to the irregularities present in the component and abrupt changes in the cross-section. In the present analysis, ANSYS has been employed to find out the localized stresses in the plate. The amount of stress concentration due to irregularities in anybody is measured by the stress concentration factor (SCF) which is denoted by and given by the generalized relation:

$$\textit{Theoretical stress concentration factor} = 1 + 2(b/a) \qquad (1)$$

It is important to note here that gross area (area of the plate ignoring the hole) has been considered in this analysis. The gross area approach is helpful for the design engineer in the way that it gives the SCF due to discontinuity based on the applied load. Although the actual SCF is calculated with the help of the net cross-sectional area of the plate. The stress concentration factor for an elliptical hole in a rectangular plate is given by the following relation: Where Major radius of the elliptical hole.

According to above as the value of decreases, the value of the Stress concentration factor increases. It tends to infinity as b approaches zero.

Plate Dimensions

Finite plate details:

1) Length of plate……………….. 500 mm
2) Width of plate……………….. 150 mm

3) Thickness of plate…………...... 5 mm.

In-finite plate details:

Condition for selecting in-finite plate b/a<5

1) Length of plate………………..500 mm
2) Width of plate………………...150 mm
3) Width of hole………............10, 12, 14,16and 18 mm.

Cut-out dimension:

1) Circular cut-out…………………. 5 mm
2) Elliptical cut-out……………….. 20 mm major and 10 mm minor axis

2.1. Modelling

Modeling is the process of producing a model; a model is a representation of the construction and working of some system of interest. A model is similar as shown in Fig.2 but simpler than the system it represents. One purpose of a model is to enable the analyst to predict the effect of changes to the system. On the one hand, a model should be a close approximation to the real system and incorporate most of its salient features. On the other hand, it should not be so complex that it is impossible to understand and experiment with it. A good model is a judicious trade-off between realism and simplicity. Simulation practitioners recommend increasing the complexity of a model iteratively. An important issue in modeling is model validity. Model validation techniques include simulating the model under known input conditions and comparing model output with system output. Generally, a model intended for a simulation study is a mathematical model developed with the help of simulation software. Mathematical model classifications include deterministic (input and output variables are fixed values) or stochastic (at least one of the input or output variables is probabilistic); static (time is not taken into account) or dynamic (time-varying interactions among variables are taken into account). Typically, simulation models are stochastic and dynamic. The 3D model of the elastic rectangular plate has been created and discretized using the modeling and meshing capabilities of ANSYS. The elements used for discretization are SOLID95, which is a higher-order version of the 3-D 8-node solid element as shown in Fig.3.

SOLID45 can tolerate irregular shapes without as much loss of accuracy. SOLID95 elements are well suited to model curved boundaries. The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. The element may have any spatial orientation. It has plasticity, creep, stress stiffening, large deflection, and large strain capabilities. The model geometry and various boundary conditions are applied.

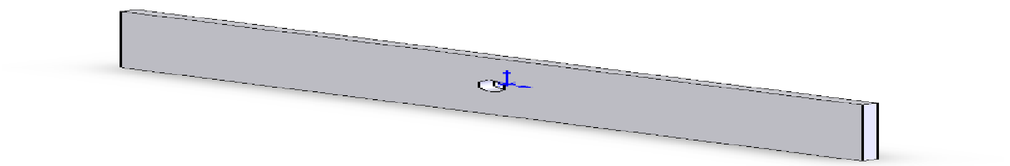


Fig.2 Geometry of plate hole with suitable dimensions in Solid works

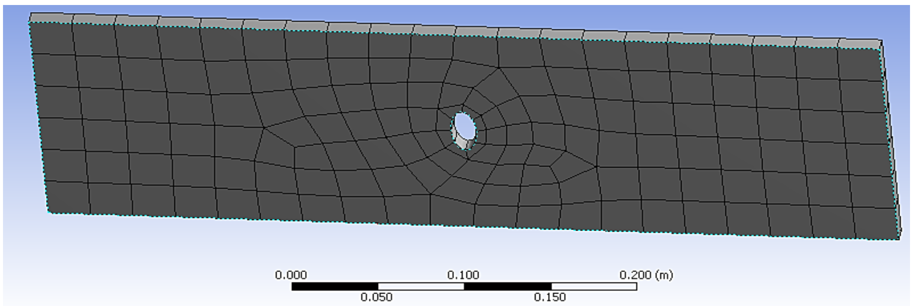


Fig.3 Meshing of plate hole with suitable dimensions in in Ansys-14

2.2. Rectangular Plate2-D Modeling

Rectangular plate modeling and meshing solutions from ANSYS offer numerous approaches in providing meshes that best meet the physics. In general, this consists of two approaches that use common tools2-D axisymmetric or planar models can be used to simplify 3-D physicsin a 2-D fashion. 2-D models can be meshed with quad meshes, quad dominant meshes, or all triangle meshes. Rectangular plate models can be used to simplify 3-D models to a set of sheets with a defined thickness. This is particularly useful for modeling sheet metal or thin structural parts. Shell parts can also be meshed with quad meshes, quad-dominant meshes, or all-triangle mesh. ANSYS meshing provides easy methods to simplify geometry to beam models or to create beam models that help users easily construct simplified models for quick analyses.

Stress analysis for trusses, beams, and other simple structures is carried out based on dramatic simplification and idealization: mass concentrated at the center of gravity, beam simplified as a line segment (same cross-section), Design is based on the calculation results of the idealized structure & a large safety factor (1.5-3) given by experience. FEA, Design geometry is a lot more complex, and the accuracy requirement is a lot higher. We need to understand the physical behaviors of a complex object (strength, heat transfer capability, fluid flow, etc.). To predict the performance and behavior of the design; to calculate the safety margin; and to identify the weakness of the design accurately and to identify the optimal design with confidence.

Theoretical Stress Concentration Factor = 1+2(b/a) (2)

3. Results and discussion

A detailed investigation of stress concentration phenomena across a variety of aspect ratios is shown in the table that has been supplied. This investigation sheds light on the complex interaction that exists between geometric parameters and the behavior of materials. One of the most important parameters that plays a role in determining how stress is distributed inside buildings is the aspect ratio, which is represented by the ratio of width to height (b/a). The stress concentration factors (Kt) associated with each aspect ratio are outlined in the Table, which reveals the amplification of stress at certain spots as a result of geometric irregularities or structural elements. These factors are derived via theoretical and simulation methodologies. A basic knowledge of how stress is amplified inside a specific geometry may be obtained via the use of theoretical stress concentration factors equation (2), which are predictions that are derived from analytical models. For example, the theoretical stress concentration factor may vary anywhere from 3.22 for an aspect ratio of 0.9 to 5 for an aspect ratio of 0.5 of the objects being considered. On the other hand, simulation-derived factors provide empirical confirmation and further insight into the behavior of real-world phenomena. For instance, a simulation stress concentration factor may range from 3.99 to 4.53 across the identical aspect ratios.

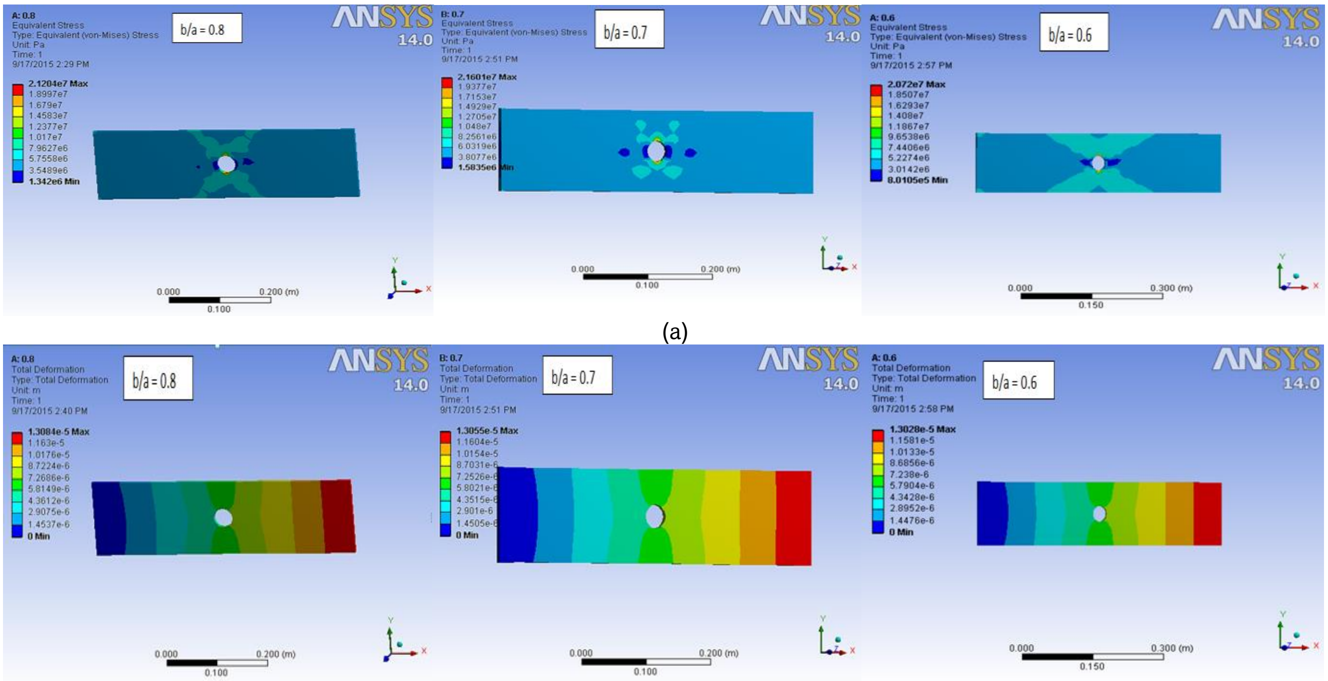
Table 1 Stress concentration factor for different aspect ratios as given below

S. No.	Aspect ratio (b/a)	Theoretical stress concentration factor (Kt)	Simulation stress concentration factor (Kt)	Von-Mises Stress (MPa)	Total deformation (mm)	% Error
1	0.9	3.22	3.99	19.99	$1.31x10^{-5}$	0.1929
2	0.8	3.5	4.29	21.20	$1.30x10^{-5}$	0.1841
3	0.7	3.85	4.32	21.60	$1.30x10^{-5}$	0.1087
4	0.6	4.33	4.14	20.72	$1.30x10^{-5}$	0.0458
5	0.5	5	4.53	22.66	$1.30x10^{-5}$	0.1037

The combined stress state at crucial sites inside the material is encapsulated by the Von-Mises stress values, which are represented in megapascals (MPa). These values reflect the complicated interaction of forces under varied aspect ratios. A variety of Von-Mises stresses, ranging from 19.99 MPa to 22.66 MPa, are shown in the table. These stresses correlate to certain aspect ratios. The total deformation, which is measured in millimeters, is a measurable indication of how the structure reacts to the loads that are given to it. It is able to record the degree of material displacement and deformation, which provides essential insights about the structure's integrity as well as its performance. According to the data that has been presented, the total distortion may range anywhere from $1.31x10^{-5}$ mm to $1.30x10^{-5}$ mm across all of the different aspect ratios.

Furthermore, the percentage error that exists between the outcomes of the simulation and the theoretical model sheds light on the accuracy of computational approaches in anticipating conditions that include stress concentration. The existence of this disparity draws attention to the complexities that are inherent in the simulation of complicated mechanical phenomena and emphasizes the need of developing computational models so that they are more in line with theoretical expectations. Percentage errors ranging from 0.0458% to 0.1929% are included in the table, which reflects the fact that there is a wide range of accuracy across the various aspect ratios involved. For the purpose of providing engineers and designers who are struggling with structural optimization and performance improvement with significant insights, **Table 1** presents a detailed study of stress concentration across a variety of aspect ratios. This dataset makes it easier to make educated decisions in the field of structural engineering by shedding light on the intricate link that exists between geometry, stress distribution, and material reaction. As a result, it paves the way for designs that are more robust and durable.

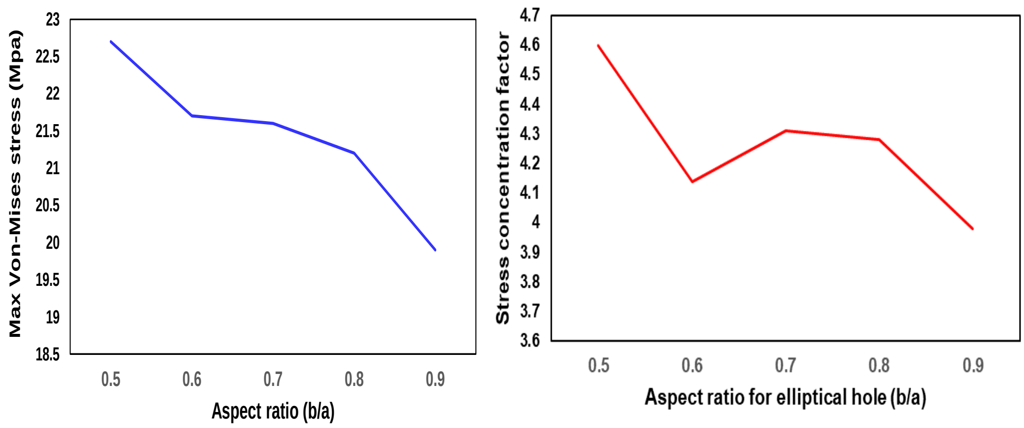
In this section, Maximum Von-Mises stress and displacement for different values of b/a (aspect ratio) of the central elliptical hole in the elastic plate are determined with the help of ANSYS. The results are displayed with the help of graphs, in Fig. 4(a) & 4(b).



(b)

Fig.4 (a) Effect of aspect ratio of elliptical hole on Von-Mises stress for b/a=0.8,0.7,0.6 & (b) Effect of aspect ratio of elliptical hole on deformation 0.8, 0.7, 0.6

The FE results of Fig. 5 (a) and (b) for SCF at various normalized main aspect radius ratios (b/a) for a plate with a central circular hole were compared for the SCFs in a thin, elliptical plate with a central hole. It is observed that the findings of the stress concentration variables are significantly influenced by the plate height to width ratio (b/a). The largest discrepancy between the plate height to width equation results and the findings of the created FE models. The largest difference in SCF decreased from about when b/a was increased from 0.5 to 0.9 in comparison to the results of the slandered equation. As per below graphs showing increasing aspect ratio the von misses stress Decreases, similarly stress concentration factor decreases.



(a) (b)

Fig. 5 Graph for (a) Von –Mises stress vs Aspect ratio (b/a) (b) graph for simulation of SCF Vs Aspect ratio of elliptical hole (b/a)

4. Conclusion

In the present study, the ANSYS was employed as a suitable way of determining stresses induced in any body and Its obtained results are not very accurate as compared to the analytical results but they can be used for the simulation of complex geometries. As compared to the analytical method which only gives the numerical value of stress, ANSYS gives a more intuitive feel to the designer by displaying stress contours throughout the plate. As the aspect ratio of the elliptical hole is increased to 1 (keeping the major diameter constant), the SCF decreases to a minimum of 3.223 (for gross cross-sectional area), which is very close to the analytical result of 3 for a circular hole. Its expected, the maximum deflection occurs when the aspect ratio of elliptical hole is 1i. e., when the ellipse is a circle. The maximum stress concentration occurs at the corners of elliptical hole in all the cases.

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